UV completions of TT perturbations of CFT

(And something about the Cosmological Constant)

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Celebration of Fedya Smirnov's major contributions to Integrable QFT
Paris, October 2023

(Based on work done in the past 2 years, published in JHEP)

The problem of UV completion of QFT

General (understood) properties of RG

Action:

$$S = S_{CFT} + \sum_i g_i \int d^2x \; \underbrace{\mathcal{O}_i(x)}_{\text{relevant and irrelevant ops.}}$$

- A low energies (IR) we ignore irrel. ops.
- In UV the irrelevant operators <u>dominate</u>.

Fundamental questions:

  * Is the UV well-defined and finite?
  * Can we reconstruct UV theory from <u>limited</u> data in IR?

Central to:

  * Beyond Standard model
  * Quantum Gravity (Asymptotic safety)

We limit the scope:

- 2d
- require integrability
- leading irrel. = $T\bar{T}$ of dim = 4

$T, \bar{T}$ ~ stress energy tensor.

$$S = S_{CFT} + \sum_{s=1}^{\infty} \alpha_s \int d^2x \, [T\bar{T}]_s$$

$[T\bar{T}]_1 = T\bar{T}$

$[T\bar{T}]_{s>1}$ from Smirnov-Zamolodchikov.
with dim $= 2(s+1)$.

s = odd integer.

Probe: Ground state energy on circle of circumference $R = 1/T$

$$E(R) = \frac{-\pi}{6} \frac{c(mR)}{R} \qquad r = MR.$$

$c(mR)$ = effective Virasoro central charge.

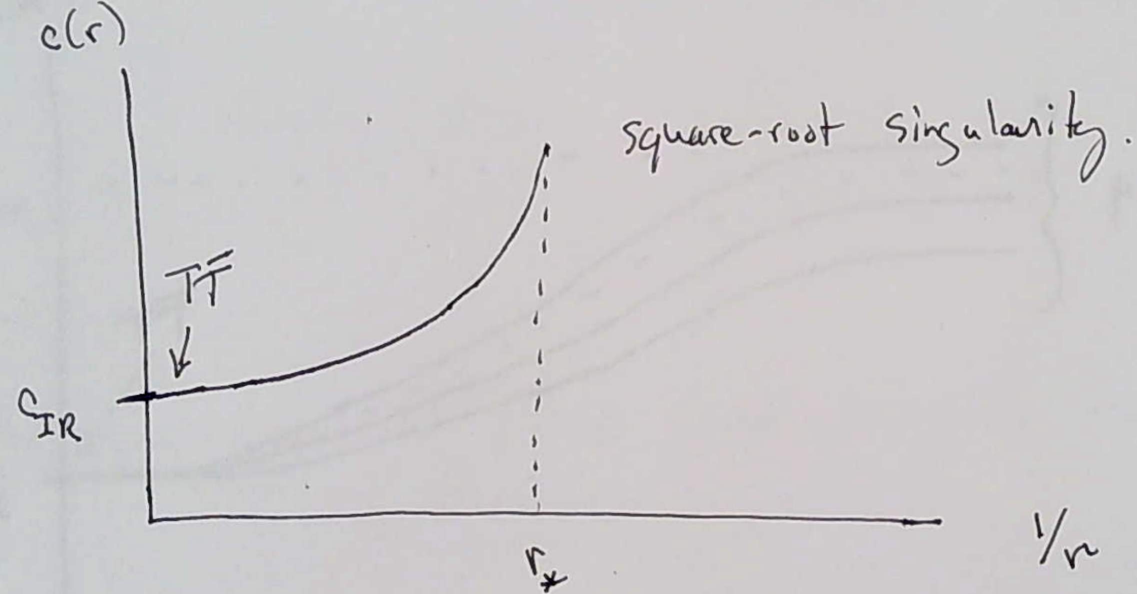
IR limit: $r \to \infty$

UV " $r \to 0$

Pure $[T\bar{T}]_1$ is universal. $(\alpha_{s>1} = 0)$

- Smirnov-Zamo, Cavaglià et. al.

$$c(r) = \frac{2\, c_{IR}}{1 + \sqrt{1 - \frac{2\pi}{3} h\, c_{IR}}} \qquad h = -\frac{\alpha}{R^2}$$



r_* = smallest possible distance.

Interpretations:

1.) String theory applications. r_* = string scale
  - Hagedorn transition
  All OK!

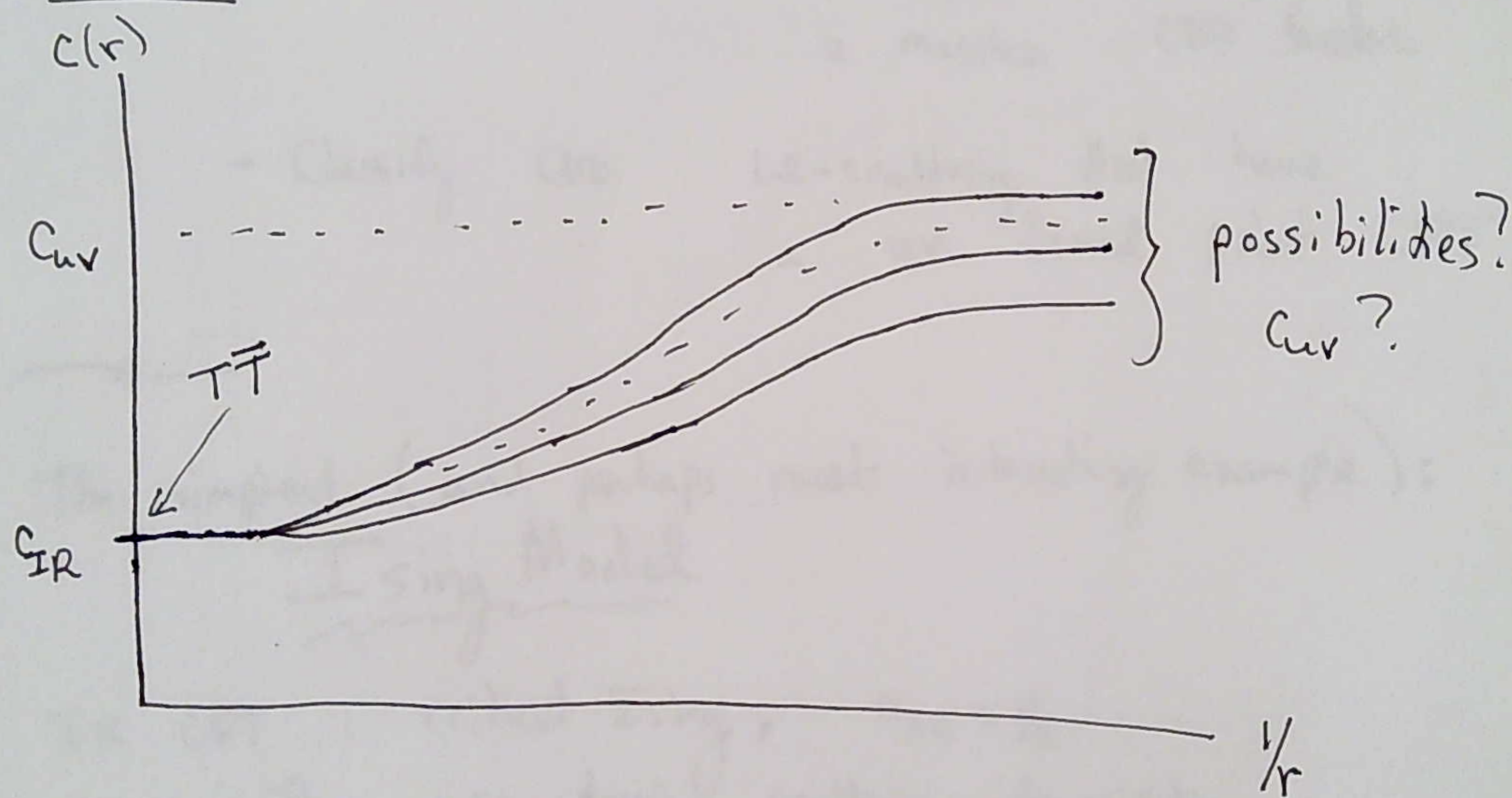
2.) QFT: pert. by irrel. ops is sick!

Our work:

Pose: • can we tune higher α_s to cure the singularity?

• can we classify the possible UV completions?

Picture.



C-theorem

• flow from UV to IR is "irreversible"
i.e. lose degrees of freedom.

• c(r) decreases as r increases.

My recent work: (recently with Changrim Ahn)

- use TBA to calculate $c(mR)$.
- Spectrum: massless Left and Right moving particles the depend on IR CFT.
- $[T\bar{T}]_s$ only affects LR scattering. (a massless CDD factor)
- Classify CDD LR-scattering that have a UV fixed point CFT

The simplest (and perhaps most interesting example):

Ising Model

IR CFT : critical Ising, $c_{IR} = 1/2$

- There are two scattering descriptions of critical Ising:

(i) "energy" spectrum : Majorana fermion.

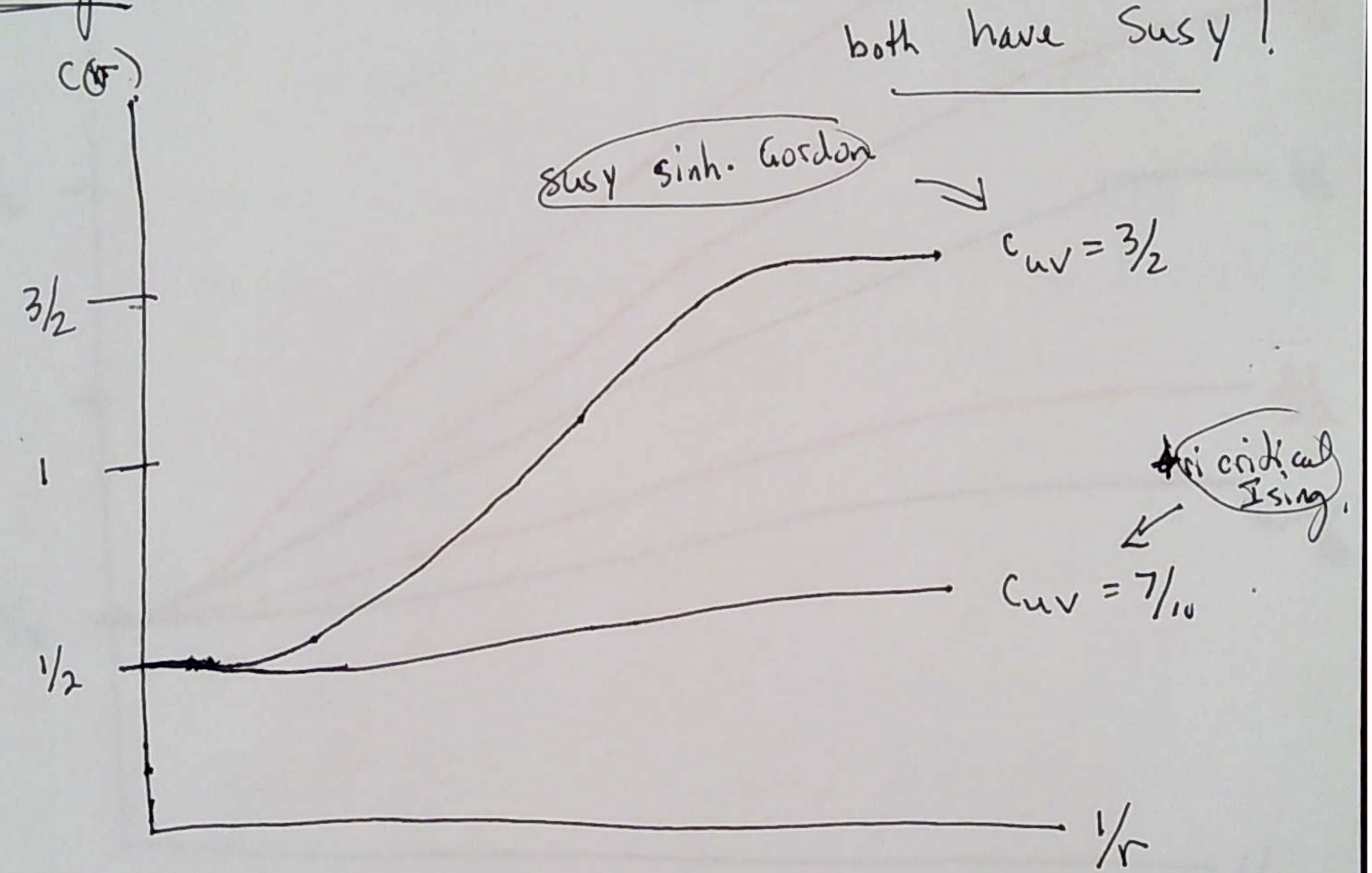
(ii). "magnetic" spectrum : Ising pert. by spin field.

A. Zamodichikov : 8 particles related to E_8

Results thus far.

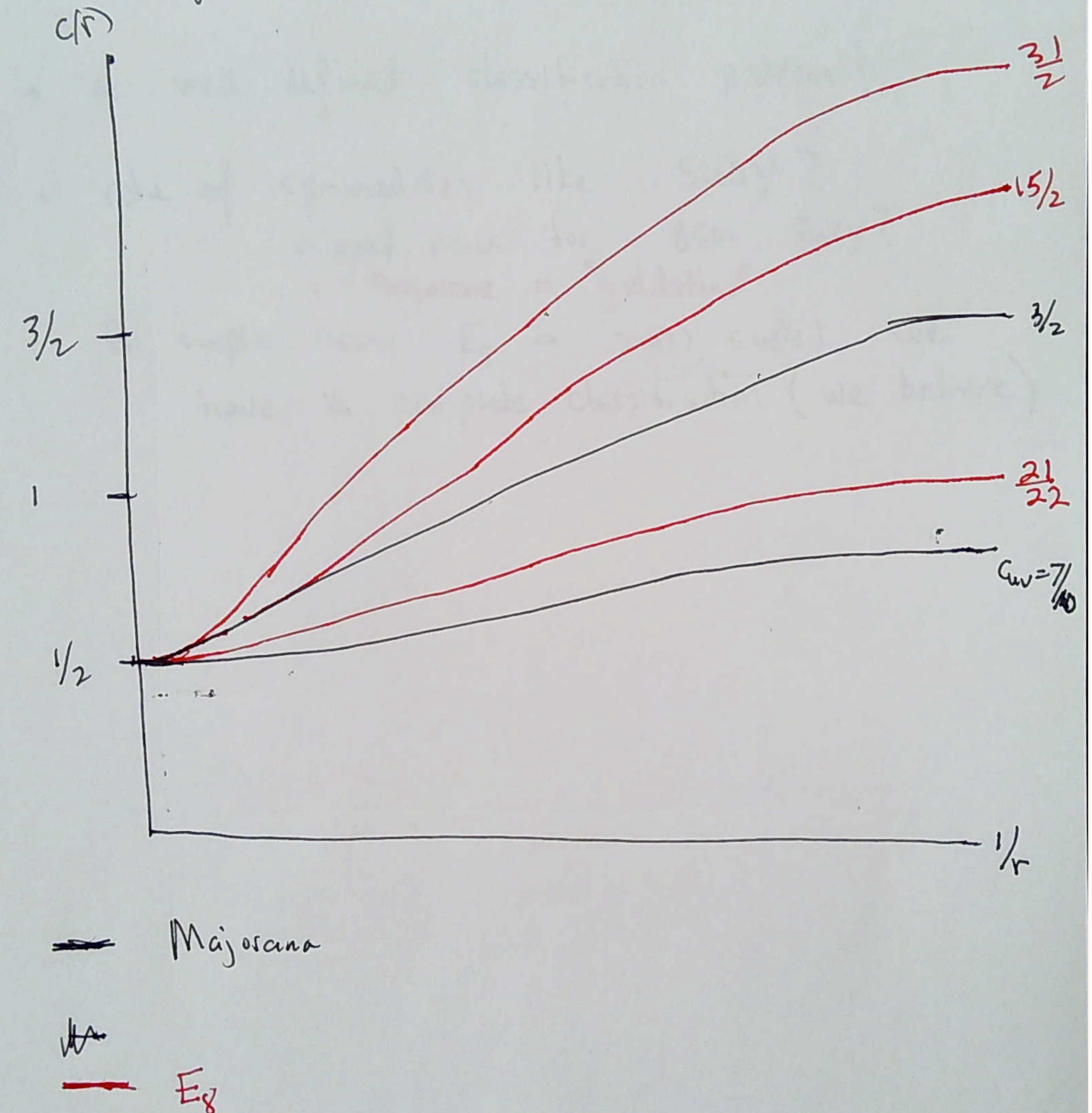
- 5 UV completions, probably more.

Ising.



- Majorana spectrum

Including E_8-cases.



Results for su(4): only 3 particles

n's ~ # CDD factors

x's ~ "plateaux" values of TBA solution

(n_1, n_2, n_3)	$(\hat{x}_1 = \hat{x}_3,\ \hat{x}_2)$	$c_{\rm UV}$	${\rm cft}_{\rm UV}$
$(0,0,0)$	$(2,3)$	$c_{\rm UV} = c_{\rm IR} = 1$	${\rm cft}_{\rm UV} = {\rm cft}_{\rm IR} = [{\rm su}(4)]_1$
$(2,0,0)$	$(1.2469, 4.0489)$	$\frac{9}{7}$	?
$(2,4,0)$ $(0,2,1)$	$(\frac{\sqrt{5}+1}{2}, \frac{\sqrt{5}+1}{2})$ "	$\frac{7}{5}$ "	?
$(1,2,1)$	$(1.2469, 1.8019)$	$\frac{11}{7}$	$[{\rm su}(4)]_2$ (minimal)
$(0,0,2)$ $(1,4,1)$	$(1, \frac{\sqrt{5}+1}{2})$ $(\frac{\sqrt{5}+1}{2}, \frac{\sqrt{5}-1}{2})$	$\frac{9}{5}$ "	${\rm su}(4)_2/{\rm su}(3)_1$?
$(2,2,1)$	$(0.8019, 2.2469)$	$\frac{13}{7}$	?
$(0,2,2)$ $(2,4,1)$	$(1,1)$ "	2 "	${\rm Pf}_2$ (diagonal)
$(1,0,2)$ $(0,3,2)$	$(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2})$ $(1, \frac{\sqrt{5}-1}{2})$	$\frac{11}{5}$ "	?
$(1,2,2)$	$(\frac{\sqrt{5}-1}{2}, 1)$	$\frac{12}{5}$	?
$(1,3,2)$	$(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}-1}{2})$	$\frac{13}{5}$	?
$(2,2,2)$	$(0,1)$	4	${\rm su}(4)_2/{\rm su}(2)_1 = {\rm su}(4)_2/u(1)$?
$(2,4,2)$	$(0,0)$	5	${\rm su}(4)_2$ (saturated)

Summary.

- a well defined classification problem!
- role of symmetries like SUSY?
  - good news for BSM SUSY?
  - Majorana = "goldstino"
- for simpler cases $E_8 \to su(3), su(4)$ we have a complete classification (we believe).

How this led to new perspectives on the Cosmological Constant Problem.....

The Thermodynamic Bethe Ansatz calculates the ground state energy on a circle of circumference R, from which we can define an effective central charge c(r), r = mR

$$E(R) = -\frac{\pi}{6} \frac{c(mR)}{R}$$

In an attempt to determine the UV fixed points, we wished to compare with conformal perturbation theory.

$$\lim_{r \to 0} c(r) = c_{\rm UV} + c_{\rm bulk} + c_{\rm pert}.$$

Parameterize: $$c_{\rm bulk} = -\frac{3r^2}{\pi \mathfrak{g}}$$ (The r-squared term)

The c-bulk term is difficult to “subtract” in order to access c-pert

As it turns out: the c-bulk term is nothing other than the 2d analog of The Cosmological Constant!

One can show, when R = 0: $\langle T_{\mu\nu} \rangle_0 = -\rho_{\rm vac}\, g_{\mu\nu}$

$$c_{\rm bulk} = -\frac{3r^2}{\pi \mathfrak{g}} \quad \Longrightarrow \quad \rho_{\rm vac} = -\frac{m^2}{2\mathfrak{g}}.$$

For many particles, m = mass of lightest particle, g = generalized coupling.
For free theories g=0 and the CC diverges. Interactions can render it finite.

The generalized coupling “g” can be computed directly from the S-matrix.

The original CCP ala Weinberg

Consider a free scalar field of mass m. Then naively:

$$\rho_{\rm vac} = 2 \int_0^{\Lambda} \frac{dk}{2\pi} \; \tfrac{1}{2} \sqrt{k^2 + m^2} \approx \; \frac{\Lambda^2}{4\pi} + \frac{m^2}{4\pi} \log(2\Lambda/m).$$

The still often quoted discrepancy of 120 orders of magnitude comes from the same calculation
In 4 spacetime dimensions, with the cut-off equal to the Planck scale.

This is nonsense. And 2d integrable models give strong hints on how to fix it. For one thing, it Requires Interactions.

Illustrative example 1: sinh-Gordon model

$$\mathcal{S} = \int d^2x \left(\frac{1}{8\pi} (\partial_\mu \phi \, \partial^\mu \phi) + 2\mu \cosh(\sqrt{2}\, b\phi) \right)$$

Using important results due to Alyosha Zamolodchikov, one can show:

$$\rho_{\rm vac} = \frac{m^2}{8 \sin \pi\gamma}. \qquad\qquad \gamma = b^2/(1+b^2).$$

For small coupling b to 0:

$$\rho_{\rm vac} \approx \frac{m^2}{8\pi b^2}.$$

Indeed it diverges!

Illustrative example 2: sine-Gordon model.

This case is richer because of the complicated spectrum of particles, solitons, Breathers etc.

$$\mathcal{S} = \int d^2x \left(\frac{1}{8\pi} (\partial_\mu \phi \, \partial^\mu \phi) + 2\mu \cos(\sqrt{2}\, \widehat{\beta} \phi) \right)$$

In this case: $$\rho_{\rm vac} = -\frac{m_s^2}{4} \tan\left(\frac{\pi \widehat{\beta}^2}{2(1-\widehat{\beta}^2)} \right)$$

Non-trivial check: at the special points (p even): $\widehat{\beta}^2 = p/(p+1)$

The quantum affine symmetry of the model corresponds

To a (fractional) supersymmetry, with charges of spin 1/p.

p= 2 is ordinary N=2 susy. The CC should vanish.

It does.

for $p = 4$, $Q^4 = H$, where $Q^4 = Q_+ Q_- Q_+ Q_- + \ldots.$ (Long ago with Vafa)

Note: no freedom to fine-tune a constant shift of the potential

Towards 4 spacetime dimensions…….

$\lambda\phi^4$ **THEORY IN d SPACETIME DIMENSIONS.**

$$\mathcal{S} = \int d^d x \left(\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4!} \phi^4 \right)$$

In a semi-classical approximation for small coupling lambda,

$$\rho_{\rm vac} \propto \frac{m^d}{\lambda}$$

And: in 2d, this reproduces the exact sinh-Gordon result at small coupling.

Something intriguing in 4d

Let us propose: $$\rho_{\text{vac}} = \frac{m_1^4}{2\mathfrak{g}} \frac{c^5}{\hbar^3}$$

Astrophysical measurements: $$\rho_{\text{vac}} \approx 10^{-9} \, \frac{\text{Joule}}{\text{meter}^3}$$

If we assume $\mathfrak{g} \approx 1$, the mass of the lightest particle is $m_1 = 0.003 \, \text{eV}$

This is precisely proposed neutrino masses!

The Swampland……

Roughly: the Swampland are theories that do not have a stringy UV completion

Using these ideas M. Montero, C. Vafa, T. Van Riet and G. Venken,

Very recently proposed (around the same time) using charged particles
and Black Holes:

$$\rho_{\rm vac} < \frac{m^4}{2e^2}$$

A good problem for Fedya:

How to calculate the vacuum energy density without the TBA, but only from Form Factors?

Thank you Fedya

for our friendship,

Collaboration, and many

Contributions to the subject!